

Brane preheating

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We study brane-world preheating in massive chaotic inflationary scenario where scalar fields are confined on the 3-brane. Assuming that quadratic contribution in energy densities dominates the Hubble expansion rate during preheating, the amplitude of inflaton decreases slowly relative to the standard dust-dominated case. This leads to an efficient production of χ particles via nonperturbative decay of inflaton even if its coupling is of order $g = 10^{-5}$. We also discuss massive particle creation heavier than inflaton, which may play important roles for the baryo- and leptogenesis scenarios.

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I. INTRODUCTION

Recent studies of superstring or M-theory predict that our universe may consist of a brane world embedded in a higher-dimensional spacetime [1–3]. In particular, it was shown that the 10-dimensional $E_8 \times E_8$ heterotic string theory, which is a strong candidate to describe our real world, is equivalent to an 11-dimensional M-theory compactified to $\mathbf{M}^{10} \times \mathbf{S}^1/Z_2$ [2]. Then the 10-dimensional spacetime is expected to be compactified into $\mathbf{M}^4 \times \mathbf{CY}^6$. Therein the standard model particles are confined on the 3-dimensional brane. On the other hand, gravitons propagate in the full spacetime.

In conventional Kaluza-Klein theories, extra dimensions are compactified on some manifolds in order to obtain 4-dimensional effective gravity theories. In contrast, Randall and Sundrum [4] recently proposed a new type of compactification, in which extra dimension is not compact but gravity is effectively 3-dimensional. Their model indicates that we are living on the 3-brane with positive tension embedded in 5-dimensional anti de-Sitter bulk spacetime. This idea invokes a particular interest for the evolution of the early universe, which may exhibit some deviations from the standard cosmology by the modifications of 4-dimensional Einstein equations on the brane [5,6].

In this respect, cosmological inflation was investigated by several authors [7,8] in the context of the brane-world scenario where scalar fields are confined on the 3-brane. The effect of the brane in higher dimensions induces extra terms in 4-dimensional effective gravity especially for high energies, e.g., a quadratic term of energy density, which leads to the larger amount of inflation relative to standard inflationary scenarios.

If the modifications due to extra dimensions persist in the post inflationary phase, this may nontrivially affect on the process of reheating. In the early stage of reheating, particles coupled to inflaton are efficiently cre-

ated by the nonperturbative and nonequilibrium process which is called preheating [9–12]. Since the particle production during preheating is sensitive to the background evolution [10], it is expected that the standard picture of preheating will be altered due to the modifications in Einstein equations on the brane. In fact we will show that the brane-world preheating broadens the parameter ranges of coupling constants where particles are efficiently produced.

II. BRANE INFLATION

In the brane world scenario by Randall-Sundrum [4], where gravity as well as matter field are effectively confined on the 3-brane in the 5-dimensional spacetime, the 5-dimensional Einstein equations can be recast in the following gravitational equations on the brane [6]:

$${}^{(4)}G_{\mu\nu} = -\Lambda_4 g_{\mu\nu} + \frac{8\pi}{M_{\text{pl}}^2} T_{\mu\nu} + \frac{1}{M_5^2} \pi_{\mu\nu} - E_{\mu\nu}, \quad (2.1)$$

where $T_{\mu\nu}$ and $\pi_{\mu\nu}$ represent the energy-momentum tensor on the brane and the quadratic term in $T_{\mu\nu}$, respectively. $E_{\mu\nu}$ is a part of 5-dimensional Weyl tensor, which carries the information in the bulk. Note that the four- and five-dimensional gravitational constants, M_{pl} and M_5 , are related with the 3-brane tension, λ , as $\lambda = 48\pi M_5^6/M_{\text{pl}}^2$.^{*} Here the 4-dimensional cosmological constant Λ_4 is assumed to be zero.

Let us analyze the gravitational equation (2.1) in cosmological contexts. Adopting a flat Friedmann-

^{*}Our notation of the 5-dimensional Planck mass, M_5 , does not include the $\sqrt{8\pi}$ factor, which is different from the definition in Ref. [7].

Robertson-Walker (FRW) metric as a background space-time on the brane, we find the following Friedmann equation [5–7]

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3M_{\text{pl}}^2} \rho \left(1 + \frac{\rho}{2\lambda}\right), \quad (2.2)$$

where a and ρ are the scale factor and the energy density of the matter on the brane, respectively. We have ignored the so-called “dark” radiation $E_{\mu\nu}$, which rapidly decreases as $\sim a^{-4}$ during inflation even if it exists.

At high energies the ρ^2 term is expected to play important roles to determine the evolution of the Universe. Hereafter we will assume that the quadratic term dominates during inflation and preheating, i.e., the condition, $\rho \gtrsim 2\lambda$, holds until the end of preheating. The inflaton field confined on the brane satisfies the Klein-Gordon equation,

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0. \quad (2.3)$$

Note that during inflation the quadratic contribution in Eq. (2.2) increases the Hubble expansion rate, which makes the evolution of inflaton slower by Eq. (2.3). Another Einstein equation in the quadratic-term dominant stage yields

$$\frac{\ddot{a}}{a} = \frac{\pi}{3M_{\text{pl}}^2\lambda} (\dot{\phi}^2 + 2V) (2V - 5\dot{\phi}^2). \quad (2.4)$$

This implies that inflation ends around $2V \simeq 5\dot{\phi}^2$. In the massive chaotic inflationary scenario with a potential, $V(\phi) = \frac{1}{2}m^2\phi^2$, this condition gives

$$\phi_F \simeq 3 \left(\frac{M_5}{m}\right)^{1/2} M_5, \quad (2.5)$$

where we used the slow-roll condition, $\dot{\phi} \simeq -m^2\phi/3H \simeq -4M_5^3/\phi$ in Eq. (2.3).

The inflaton mass, m , should be constrained by comparing the amplitude of density perturbations produced during inflation with the CMB anisotropy. This is typically done by evaluating the curvature perturbation, ζ , around the 55 e-foldings before the end of inflation, if ζ is conserved on large scales. It was shown in Ref. [13] that the conservation of ζ holds for adiabatic perturbations irrespective of the form of gravitational equations by considering the local conservation of the energy-momentum tensor. In the presence of a second scalar field, χ , curvature perturbations include isocurvature (entropy) perturbations [13,14], which provides the source of nonadiabatic pressure perturbations. However, as long as χ does not exhibit strong growth during inflation as in the nonminimally coupled case [15], ζ remains conserved even in the multi-field case [16]. In fact, in the massive chaotic inflationary scenario with the interaction $\frac{1}{2}g^2\phi^2\chi^2$, χ is suppressed during inflation for large couplings g required for the χ particle production during preheating [17], which

ensures the conservation of ζ on large scales. Although we will use this result here, the Weyl tensor $E_{\mu\nu}$ will bring the 5-dimensional information into the brane, which may alter the result [18]. More careful analysis will be required regarding the evolution of ζ during inflation [19].

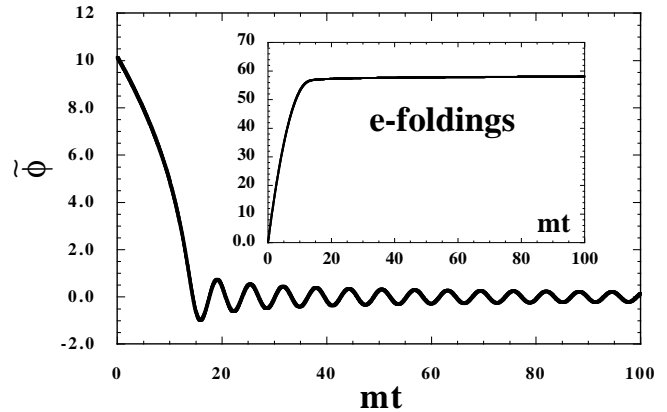


FIG. 1. The evolution of the inflaton condensate, $\tilde{\phi} = (m/M_5)^{1/2}\phi/M_5$, during inflation and reheating. We start integrating with the initial value in Eq. (2.6). Inflation ends around $\tilde{\phi} \simeq 3$ as estimated by Eq. (2.5). The system enters the reheating stage around $\tilde{\phi} \simeq 1$. **Inset:** The evolution of the number of e-foldings, $N = \int_{t_*}^{t_F} H dt$. We find that 55 e-foldings are achieved during inflation.

Calculating the number of e-foldings: $N \equiv \int_{t_*}^{t_F} H dt$, and setting $N = 55$, we obtain the value of inflaton when scales relevant for the CMB anisotropy crossed outside the Hubble radius,

$$\phi_* \simeq 10 \left(\frac{M_5}{m}\right)^{1/2} M_5. \quad (2.6)$$

The spectra of density perturbations at horizon re-entry can be evaluated as $\delta_H = (2/5)|\zeta|_{k=aH} = (2/5)H\delta\phi/\dot{\phi}|_{k=aH}$, where the subscript, $k = aH$, denotes the values at first horizon crossing. Making use of the relation $\delta\phi \simeq H/2\pi$ at first horizon crossing[†], we find

$$\delta_H \simeq \frac{1}{(24\sqrt{5}\pi)^2} \left(\frac{m}{M_5}\right)^4 \left(\frac{\phi_*}{M_5}\right)^5. \quad (2.7)$$

The COBE normalization gives $\delta_H \simeq 2 \times 10^{-5}$, which constrains the mass of inflaton as

$$m/M_5 \simeq 1.4 \times 10^{-4}. \quad (2.8)$$

The ratio of Eq. (2.8) is crucial to determine the efficiency of parametric resonance during preheating. We

[†]While the formula used here is based on the assumption of the Bunch-Davies vacuum, the present expansion law is different from the de Sitter one, which may change the expression of δ_H .

should caution again that the result of Eq. (2.8) could be modified by a careful study of the quantum theory in the brane-world inflation and the evolution of cosmological perturbations in the 5-dimensional Einstein equations [18,19].

We have numerically solved the background equations (2.2) and (2.3) with the initial value (2.6), and confirmed that analytic estimations are in good agreement with numerical results (see Fig. 1).

III. BRANE PREHEATING

Let us investigate a two-field model where the second scalar field, χ , is coupled to massive inflaton, ϕ , with a four-leg interaction, $\frac{1}{2}g^2\phi^2\chi^2$:

$$V(\phi, \chi) = \frac{1}{2}m^2\phi^2 + \frac{1}{2}g^2\phi^2\chi^2 + \frac{1}{2}m_\chi^2\chi^2, \quad (3.1)$$

where m_χ is a mass of the χ field. We consider perturbations of scalar fields on the 3-brane in the flat FRW background and implement backreaction effects of produced particles at second order [10].

Decomposing the scalar fields into homogeneous parts and fluctuations as $\phi(t, \mathbf{x}) \rightarrow \phi(t) + \delta\phi(t, \mathbf{x})$ and $\chi(t, \mathbf{x}) \rightarrow \chi(t) + \delta\chi(t, \mathbf{x})$, the equation for the homogeneous part of inflaton can be written as

$$\ddot{\phi} + 3H\dot{\phi} + (m^2 + g^2\langle\chi^2\rangle)\phi = 0, \quad (3.2)$$

where $\langle\chi^2\rangle$ is the expectation value of χ^2 , which works as backreaction at the final stage of preheating. Neglecting the contribution of inflaton fluctuations which can be negligible in the present model unless rescattering effects are taken into account, each Fourier mode of the $\delta\chi_k$ fluctuation satisfies

$$\delta\ddot{\chi}_k + 3H\delta\dot{\chi}_k + \left[\frac{k^2}{a^2} + g^2\phi^2(t) + m_\chi^2\right]\delta\chi_k = 0. \quad (3.3)$$

The oscillation of the inflaton condensate leads to parametric amplification of the $\delta\chi_k$ fluctuation, whose variance is defined by

$$\langle\delta\chi^2\rangle = \frac{1}{2\pi^2} \int k^2 |\delta\chi_k|^2 dk. \quad (3.4)$$

Note that the equation for $\chi(t)$ is obtained by substituting $k = 0$ in Eq. (3.3), whose evolution is similar to the long wave $\delta\chi_k$ modes.

Assuming that the linear term of ρ in Eq. (2.2) can be neglected relative to the quadratic term during the whole stage of preheating, the Hubble parameter satisfies

$$H = \frac{1}{6M_5^3} \left[\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}m^2\phi^2 + \frac{1}{2}\dot{\chi}^2 + \frac{1}{2}m_\chi^2\chi^2 + \frac{1}{2}g^2\phi^2\chi^2 + \rho_\phi + \rho_\chi \right], \quad (3.5)$$

where ρ_ϕ and ρ_χ are the energy densities of produced particles. In the stage where contributions of χ are negligible in Eqs. (3.2) and (3.5), making use of the time averaged relation, $\langle\frac{1}{2}\dot{\phi}^2\rangle_T = \langle V(\phi)\rangle_T$, gives the following approximate solutions for the scale factor and the background inflaton field:

$$a = a_i \left(\frac{t}{t_i}\right)^{1/3}, \quad \phi(t) = \Phi(t) \sin mt, \quad (3.6)$$

where the amplitude, $\Phi(t)$, is

$$\Phi(t) = 2 \left(\frac{M_5}{m}\right)^{3/2} \sqrt{\frac{m}{t}}. \quad (3.7)$$

This evolution is different from the standard results in the dust-dominated preheating scenario, $a \propto t^{2/3}$ and $\Phi(t) \propto t^{-2}$ [10]. Particle creations in an expanding universe are strongly affected by the change of background evolution.

Introducing a new variable, $\delta X_k = a^{3/2}\delta\chi_k$, and neglecting a curvature term in the frequency of δX_k which is unimportant during reheating, we find that Eq. (3.3) is reduced to the Mathieu equation

$$\frac{d^2}{dz^2}\delta X_k + (A_k - 2q \cos 2z)\delta X_k = 0, \quad (3.8)$$

where $z = mt$ and

$$A_k = 2q + \frac{k^2}{m^2 a^2} + \frac{m_\chi^2}{m^2}, \quad q = \frac{g^2\Phi^2(t)}{4m^2} = g^2 \left(\frac{M_5}{m}\right)^3 \frac{1}{z}. \quad (3.9)$$

The efficiency of resonance strongly depends upon the initial value of q ($= q_i$). In the standard preheating scenario where q decreases as $q \sim t^{-2}$, we require the coupling, $g \gtrsim 10^{-4}$ which corresponds to $q_i \gtrsim 100$, for the relevant growth of $\delta\chi_k$ fluctuations [10].

In contrast, in the brane-world preheating, q evolves more slowly as $q \sim t^{-1}$, which indicates that large q_i much greater than unity is not required for χ particle productions to occur. This has some analogies with the nonminimally coupled χ field in the R^2 inflation model where slow decrease of the oscillating scalar curvature ($R \propto t^{-1}$) leads to strong preheating [20]. The ratio m/M_5 is an important factor to determine the strength of resonance. Adopting the value of Eq. (2.8) which comes from the CMB constraints, one obtains

$$q \simeq 0.36 (10^6 g)^2 \frac{1}{z}. \quad (3.10)$$

The coherent oscillation of inflaton turns on around $z \simeq \pi/2$, which yields the relation, $q_i \simeq (10^6 g)^2/5$. When $m_\chi = 0$, we find numerically that $\delta\chi_k$ fluctuations exhibit exponential increase for $g \gtrsim 5 \times 10^{-6}$, corresponding to $q_i \gtrsim 5$.

Let us consider concrete cases with a massless χ field. When $g = 5 \times 10^{-6}$, the variance $\langle \delta\chi^2 \rangle$ increases at the initial stage, but its growth stops around $mt \simeq 35$ (see Fig. 2). Although χ particles are created while $\delta\chi_k$ fluctuations pass through instability bands with the decrease of q due to cosmic expansion, no creation occurs once q drops down to $1/4 \sim 1/3$, since no instability bands exist there [10]. Actually, substituting $z = 35$ for Eq. (3.10) gives the value, $q \simeq 0.26$, which coincides well with the above estimation.

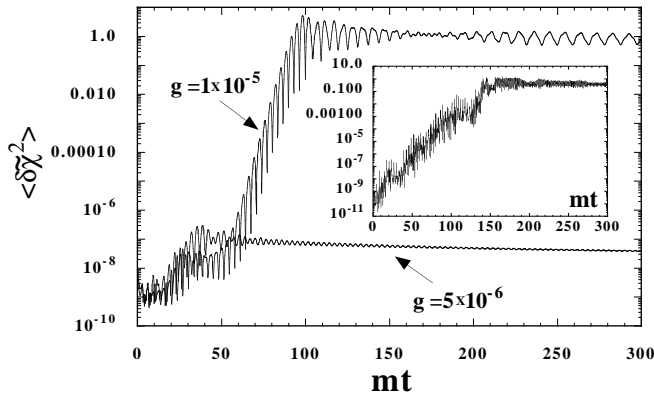


FIG. 2. The evolution of the variance, $\langle \delta\chi^2 \rangle \equiv \langle \delta\chi^2 \rangle / M_5^2$, during preheating for $g = 5 \times 10^{-6}$ and $g = 1 \times 10^{-5}$ with $m_\chi = 0$. While resonance soon terminates at the initial stage for $g = 5 \times 10^{-6}$, χ particles are sufficiently produced for $g = 1 \times 10^{-5}$. **Inset:** $\langle \delta\chi^2 \rangle$ vs t for the case of $g = 2 \times 10^{-3}$, $m_\chi = 10m$. The χ particle whose mass is greater than the inflaton mass, m , can be sufficiently created for $g > 2 \times 10^{-5}$.

With the increase of g , the duration during which the χ field stays in instability bands gets longer. In Fig. 2 we plot the evolution of $\langle \delta\chi^2 \rangle$ for $g = 10^{-5}$. We find that the field reaches a short plateau around $25 \lesssim mt \lesssim 50$ after some growth in the initial stage. This corresponds to the stability region around $1 \lesssim q \lesssim 2$, after which $\langle \delta\chi^2 \rangle$ begins to grow again since the field passes through the first instability band around $1/3 \lesssim q \lesssim 1$. In this case the backreaction effect of created χ particle is marginally important at the final stage of preheating.

When $g \gtrsim 10^{-5}$, the backreaction effect plays an important role to shut off the resonance. While the growth rate of $\delta\chi_k$ fluctuations gets larger with g being increased, the final variance begins to be suppressed, since resonance becomes less efficient when the $g^2 \langle \chi^2 \rangle$ term in Eq. (3.2) grows to the order of m^2 . Numerical simulations based on the Hartree approximation imply that the final variance slowly decreases for $g \gtrsim 10^{-4}$. In order to estimate the correct size of fluctuations, however, we need to include the rescattering effect (i.e., mode-mode coupling), which becomes important when particles are sufficiently excited [11].

If the χ mass is taken into account, this generally works

as a suppression for an efficient preheating. However, as long as the mass effect does not make the χ field deviate from instability bands, massive χ particles can be resonantly amplified. For example χ particles with mass $m_\chi = m$ are created for $g \gtrsim 2 \times 10^{-5}$; and when $m_\chi = 10m$, $g \gtrsim 2 \times 10^{-3}$ (See the inset of Fig. 2). Since heavy particles whose masses are a few times greater than the inflaton mass are sufficiently produced even when $g \gtrsim 10^{-4}$, this may play an important role for the success of the GUT scale baryogenesis scenario [21,22]. It will be particularly interesting to estimate the baryon asymmetry produced in preheating, taking into account the decay of χ particles as in Ref. [22].

The spin 1/2 (and higher) quantum fields can be also produced during preheating [23,24], although the Pauli's exclusion principle restricts the occupation number as $n_k \leq 1$. Consider a massive fermion field ψ with a bare mass m_ψ , which is coupled to inflaton with a Yukawa interaction, $h\phi\bar{\psi}\psi$. Then the Dirac equation is written in the form:

$$(i\gamma^\mu \nabla_\mu - m_{\text{eff}})\psi = 0, \quad (3.11)$$

where m_{eff} is the effective mass of fermion, defined by

$$m_{\text{eff}} = m_\psi + h\phi. \quad (3.12)$$

In an expanding universe with the FRW background, fermion production takes place around the region where m_{eff} vanishes, which corresponds to the value, $\phi_* = -m_\psi/h$ [24]. Fermions are periodically created when the inflaton passes through ϕ_* , as long as the relation $m_\psi < |h\Phi|$ is satisfied. Substituting Eq. (3.6) with Eq. (3.7) for Eq. (3.12), one finds that the minimum effective mass is achieved for $mt = 3\pi/2$, yielding the upper limit of massive fermion production as

$$m_\psi \lesssim h \left(\frac{M_5}{m} \right)^{3/2} m \simeq (10^6 h) m. \quad (3.13)$$

This implies that heavy fermions much greater than the inflaton mass are easily excited unless h is small as $h \lesssim 10^{-6}$. Since the amplitude of inflaton decreases slowly relative the standard preheating, this may delay the time at which fermion creations are frozen. It will be worth investigating how massive fermions produced in brane preheating affect the leptogenesis scenario [25].

IV. CONCLUSIONS AND DISCUSSIONS

We have studied preheating in the 3-brane world in the massive chaotic inflationary scenario. While the existence of the brane in higher dimensions works to increase the amount of inflation via quadratic modifications of energy density, its effect in preheating is to make the adiabatic damping of the inflaton slower. Considering the χ field coupled to inflaton via interaction, $\frac{1}{2}g^2\phi^2\chi^2$, we

find that massless χ particles exhibit strong amplification even when the coupling g is small as $g = 5 \times 10^{-6}$, due to the modifications of the background evolution. We have also verified that massive scalar and fermionic particles heavier than inflaton can be nonperturbatively produced, which may play important roles for the baryo- and lepto-genesis scenarios.

Although we have restricted ourselves in the massive inflaton case, the self coupling inflationary scenario with an effective potential, $V(\phi) = \frac{1}{4}\lambda\phi^4 + \frac{1}{2}g^2\phi^2\chi^2$, has a different structure of resonance. In this case introducing a new variable, $\delta X_k = a\delta\chi_k$, Eq. (3.3) is reduced to the following Lamé equation in the linear stage of preheating:

$$\frac{d^2}{dx^2}\delta X_k + \left[\kappa^2 + \frac{g^2}{\lambda}\text{cn}^2\left(x, \frac{1}{\sqrt{2}}\right) \right] \delta X_k = 0, \quad (4.1)$$

where $x \equiv \sqrt{\lambda}\phi_i\eta$ and $\kappa^2 \equiv k^2/(\lambda\phi_i^2)$, with $\eta \equiv \int a^{-1}dt$ being a conformal time. The $\text{cn}(x, 1/\sqrt{2})$ is the elliptic function which is well approximated as $\text{cn}(x, 1/\sqrt{2}) \approx \cos(0.8472x)$. Since the effect of the adiabatic damping of inflaton does not appear in the frequency of δX_k , the strength of resonance is determined by the ratio, g^2/λ , which is the same as the standard scenario discussed in Ref. [26]. Despite this, the slow growth of the scale factor makes the redshift of $\delta\chi_k$ less efficient due to the relation of $\delta\chi_k = a^{-1}\delta X_k$, which will provide the larger growth of the variance, $\langle\delta\chi^2\rangle$.

The enhancement of field perturbations can lead to the growth of metric perturbations [27,28], which is particularly important for the formation of primordial black holes [29]. This requires a consistent study of cosmological perturbations including the effect of the 5-dimensional bulk, which we leave to future work.

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